Rotordynamics Analysis Overview

Featuring Analysis Capability of RAPPID™

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Vibration of Rotating Systems

- Vibration Signature can be Quite Complex in Nature

- Vibration Modeling is Frequently used to Aid Design & Development

- Vibration Model
  - Structural Characteristics
  - Structure-to-Structure Transfer Functions
  - Forcing Functions

- Vibration Signature
  - Frequency
  - Magnitude
  - Phase
Vibration Signature Contributors

• **Major Peak Sources**
  - Structural Harmonics
  - Impacts/Rubs
  - Misalignment
  - Rotor Bow
  - Unbalance
    - Mechanical
    - Hydraulic
  - Trapped Fluids
  - Unstable Rotor Whirl
  - Large Scale Flow Effects
    - Periodic
    - Unsteady
    - Unstable

• **Noise Floor Sources**
  - Acoustics
  - Light Rubs
  - Small Scale Flows Effects
    - Turbulent eddies
    - Cavitation bubbles
    - Localized reversals

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![Graph of vibration signature contributors](image)
Typical Commercial Machinery

- Stationary
  - Few Weight/Volume Restrictions
    - Low speed
    - Rigid housing
    - Sub-critical rotors
    - Compartmentalized designs
  - Oil Lubrication Systems
    - Incompressible lubricant
      - Stiffness, damping, mass coef.
    - Viscous lubricant
      - High rotor damping

- Low Energy Density
  - Pump: 0.5 Hp/lb
  - Gas Turbine: 3 Hp/lb

- Continuous Operation
  - Steady State
  - Thermal Equilibrium
  - Constant Power Levels

Minimal Unsteady Effects
Typical Rocket Engine Turbopump

- Mobile
- Extrem weight/Volume Restrictions
  - High speed
  - Flexible rotors & housings
  - Highly integrated designs
- Process Lubricated
  - Compressible lubricants
    - Transfer functions
  - Low viscosity lubricants
    - Lightly damped rotors
- Varying Ambient Conditions

- High Energy Density (~ 100 Hp/lb)
  - Use of Cryogenes
    - Wide pressure/temperature ranges
  - Steep Ramp Rates

- Short Run Durations
  - Power Level Changes
    - Steady state never achieved

Major Unsteady Effects
### Rotordynamics Model

<table>
<thead>
<tr>
<th>Elements of Vibration Model</th>
<th>Sub Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural Characteristics</td>
<td>Rotating Assemblies</td>
</tr>
<tr>
<td></td>
<td>Stationary Assemblies</td>
</tr>
<tr>
<td>Structure-to-Structure Transfer Functions</td>
<td>Fluidic Interfaces</td>
</tr>
<tr>
<td></td>
<td>Mechanical Interfaces</td>
</tr>
<tr>
<td></td>
<td>Hybrid Interfaces</td>
</tr>
<tr>
<td>Forcing Functions</td>
<td>Flow Related</td>
</tr>
<tr>
<td></td>
<td>Mechanical Related</td>
</tr>
<tr>
<td></td>
<td>Electrical/Magnetic Related</td>
</tr>
<tr>
<td></td>
<td>Controls Related</td>
</tr>
<tr>
<td></td>
<td>Rotor/Stator Interactions</td>
</tr>
<tr>
<td>Operating Geometry Changes</td>
<td>Distortions</td>
</tr>
<tr>
<td></td>
<td>Relative Displacements</td>
</tr>
</tbody>
</table>
Structural Characteristics

- **Structural Characteristics Purpose**
  - Establish Structural Compliance and Resonance Frequencies

- **Required for Rotating and Stationary Assemblies**
  - Includes Facility/Engine Effects

- **Critical Phenomena**
  - Mass
  - Inertia
  - Load Path
  - Material Properties
    - Temperature dependent

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Accurate structural characterization is critical for establishing natural frequency locations
Transfer Functions

Transfer Function Purpose

- Translate the Motion of One Structure into Forces on Another Structure
  - Motion measured in displacements, velocities, accelerations

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} =
\begin{bmatrix}
H_{xx}(\omega) & H_{xy}(\omega) \\
H_{yx}(\omega) & H_{yy}(\omega)
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
\]

Required for all Rotating and Stationary Structure Interfaces

- Typical Interfaces Include
  - Bearings, seals, dampers
  - Pump, turbine, inducer wheels
  - Splines/couplings
  - Pump out vanes
  - Rub surfaces

Accurate transfer functions characterization is critical for establishing orbit stability and natural frequency locations
Transfer Functions

• General Form of Transfer Function (H matrix)
  - H Matrix Elements are Complex
  - H Matrix Elements May Vary Non-Linearly with Frequency

\[
\begin{align*}
\begin{bmatrix} F_x \\ F_y \end{bmatrix} &= \begin{bmatrix} H_{xx}(\omega) & H_{xy}(\omega) \\ H_{yx}(\omega) & H_{yy}(\omega) \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \\
\end{align*}
\]

- Assuming an Interface Adheres to the Linearized Model Leads To:

\[
\begin{align*}
H_{xx}(\omega) &= K_{xx} + iC_{xx} \omega - M_{xx} \omega^2 \\
H_{xy}(\omega) &= K_{xy} + iC_{xy} \omega - M_{xy} \omega^2 \\
H_{yx}(\omega) &= K_{yx} + iC_{yx} \omega - M_{yx} \omega^2 \\
H_{yy}(\omega) &= K_{yy} + iC_{yy} \omega - M_{yy} \omega^2 
\end{align*}
\]
Forcing Functions

• Forcing Function Purpose
  • Excite the Rotating and Stationary Assemblies
    • Defined by magnitude, frequency, and phase

• Required for Excitation Forces Acting on the Rotating and Stationary Structures
  • Typical Excitation Forces Include
    • Rotor unbalance
    • Impacts/rubs
    • Misalignment, shaft bow, loose press fits
    • Hydraulic unbalance
    • Trapped fluid in a rotating structure
    • Steady and unsteady flow fluctuations
      • Valve induced
      • Controller induced
    • Controller imperfections
    • Rotor/stator interactions
      • Jet, vane pass, vortex shedding
Rotordynamic Analysis

• Available Analysis Types
  
  • Eigenvalue
    • Free-Free
    • Undamped Critical Speed
    • Damped Eigenvalue (Stability)

  • Forced Response (Linear)
    • Steady State

  • Forced Response (Non-Linear)
    • Transient (not covered in this information package)
Free-Free Analysis

• Required Information
  • Structural Model

• Analysis Assumptions
  • No Rotation
  • No Interconnection Forces
  • No Forcing Functions

• Analysis Results
  • Natural Frequencies
  • Mode Shapes (planar)

• Why Perform Free-Free Analysis?
  • Verify Structural Model by Comparing to Rap Test Data
Sample Rotor Model
1st Free-Free Bending Mode

ROTORDYNAMIC MODE SHAPE PLOT
Manual Example Case
Multi-Stage Compressor
ANALYSIS POINT: ROTOR SPEED = 0 rpm
NAT FREQUENCY = 17946 cpm
2\textsuperscript{nd} Free-Free Bending Mode

ROTORDYNAMIC MODE SHAPE PLOT
Manual Example Case
Multi-Stage Compressor
ANALYSIS POINT: ROTOR SPEED = 0 rpm
NAT FREQUENCY = 27886 cpm
Undamped Critical Speed Analysis

• Required Information
  • Structural Model
  • Range of Bearing Stiffness

• Analysis Assumptions
  • No Damping
  • No Cross-Coupling
  • Symmetric Rotor Supports
  • Natural Frequency Coincides with Running Speed

• Analysis Results
  • Synchronous Critical Speed as a Function of Direct Stiffness
  • Mode Shapes (circular)

• Why Perform Undamped Critical Speed Analysis?
  • If Precise Transfer Functions are not Available
Undamped Critical Speed Map
1st Synchronous Critical Speed

ROTORDYNAMIC MODE SHAPE PLOT
Manual Example Case: Multi-Stage Compressor
Eigenvalue Analysis as a Function of Rotor Speed
ANALYSIS POINT: Thrust End Bearing Stiffness = 500000 (lb/in)
CRITICAL SPEED = 5347 rpm
STATION 22 ORBIT FORWARD PRECESSION (FORWARD=RED, BACKWARD=BLUE)
2nd Synchronous Critical Speed

ROTORDYNAMIC MODE SHAPE PLOT
Manual Example Case: Multi-Stage Compressor
Eigenvalue Analysis as a Function of Rotor Speed
ANALYSIS POINT: Thrust End Bearing Stiffness = 500000 (lb/in)
CRITICAL SPEED = 12798 rpm
STATION 44 ORBIT FORWARD PRECESSION (FORWARD=RED, BACKWARD=BLUE)
Damped Eigenvalue Analysis

• Required Information
  • Structural Model
  • Transfer Functions

• Analysis Assumptions
  • No External Excitation

• Analysis Results
  • Natural Frequency Map
  • Stability Map
  • Mode Shapes (elliptical)

• Why Perform Damped Eigenvalue Analysis?
  • Provides Essential Frequency Survey to Locate Potential Synchronous and Non-Synchronous Critical Speeds
  • Provides only Steady State Assessment of Stability
Bearing Stiffness Values

ROTORDYNAMIC COEFFICIENT PLOT: Thrust end bearing

Manual Example Case: Multi-Stage Compressor

Eigenvalue Analysis as a Function of Rotor Speed

STIFFNESS (1.0E-3 lb/in)

Roter Speed (rpm)

RAPP V2.60 Run: 10/21/2008 at 10:32:42, Plot File Name = PLOT.TXT
Bearing Damping Values

ROTORDYNAMIC COEFFICIENT PLOT: Thrust end bearing

Manual Example Case: Multi-Stage Compressor

Eigenvalue Analysis as a Function of Rotor Speed
Natural Frequency Map

ROTORDYNAMIC NATURAL FREQUENCY MAP
Manual Example Case: Multi-Stage Compressor
Eigenvalue Analysis as a Function of Rotor Speed

Potential Critical Speeds Located
Stability Map

ROTORDYNAMIC STABILITY MAP
Manual Example Case: Multi-Stage Compressor
Eigenvalue Analysis as a Function of Rotor Speed

Log Dec @ Potential Critical Speeds Labeled
Mode Shape: Mode 1

ROTORDYNAMIC MODE SHAPE PLOT - MODE #1
Manual Example Case: Multi-Stage Compressor
Eigenvalue Analysis as a Function of Rotor Speed
ANALYSIS POINT: Rotor Speed = 5000 (rpm)
NAT FREQ = 3930 cpm, LOG DEC = 0.429, POTENTIAL SYNC CRIT SPEED = 3594 rpm
STATION 22 ORBIT FORWARD PRECESSION (FORWARD=RED, BACKWARD=BLUE)
Mode Shape: Mode 2

ROTORDYNAMIC MODE SHAPE PLOT - MODE #2
Manual Example Case: Multi-Stage Compressor
Eigenvalue Analysis as a Function of Rotor Speed
ANALYSIS POINT: Rotor Speed = 5000 (rpm)
NAT FREQ = 7192 cpm, LOG DEC = 0.845, POTENTIAL SYNC CRIT SPEED = 6992 rpm
STATION 22 ORBIT BACKWARD PRECESSION (FORWARD=RED, BACKWARD=BLUE)
Forced Response – Steady State

• **Required Information**
  - Structural Model
  - Transfer Functions
  - Forcing Functions

• **Analysis Assumptions**
  - Unbalance Always Modeled
  - Other Forcing Functions Modeled as Needed

• **Analysis Results**
  - Vibration Amplitude
  - Dynamic Bearing Loads
  - Deflected Rotor Shapes (elliptical)

• **Why Perform Steady State Forced Response Analysis?**
  - Locate Actual Synchronous and Non-Synchronous Critical Speeds
  - Determine Amplification Factors
  - Establish Response Shapes
Horizontal Vibration @ Bearing

X' AXIS RESPONSE, STATION 8, Thrust end bearing

Manual Example Case: Multi-Stage Compressor

Synchronous Forced Response Analysis as a Function of Rotor Speed

SYNC RESP ANALYSIS
FORCING FUNCTION
LOC AND MAG
STA  MAG  PHASE
NO.  oz-in  deg
16.  0.28  0
18.  0.28  0
20.  0.28  0
22.  0.28  0
24.  0.28  0
26.  0.28  0
28.  0.28  0
30.  0.28  0

MOUNT ANGLE = 45.00 degrees

Actual Critical Speed(s) Located

AF=2.85M=57% 3900 RPM
Vertical Vibration @ Bearing
Maximum Vibration @ Bearing

MAJOR/MINOR ORBITAL DISP., STATION 8, Thrust end bearing

Manual Example Case: Multi-Stage Compressor

Synchronous Forced Respose Analysis as a Function of Rotor Speed

SYNC RESP ANALYSIS
FORCING FUNCTION
LOC AND MAG
STA MAG PHASE
NO. oz-in deg
16 0.28 0
18 0.28 0
20 0.28 0
22 0.28 0
24 0.28 0
26 0.28 0
28 0.28 0
30 0.28 0

DISPLACEMENT (p-p) ELLIPSE ANGLE (DEG)

MAJOR AXIS MINOR AXIS RUB LIMIT = 6 mils diam

OPER RANGE

Rotor Speed (rpm)

0 2500 5000 7500 10000 12500 15000
Maximum Dynamic Bearing Load
Rotor Response Shape

ROTOR RESPONSE SHAPE

ROTOR SPEED = 5000 rpm
Manual Example Case: Multi-Stage Compressor
Synchronous Forced Response Analysis as a Function of Rotor Speed
STATION 22 ORBIT FORWARD PRECESSION (FORWARD=RED, BACKWARD=BLUE)

SYNC RESP ANALYSIS
FORCING FUNCTION
LOC AND MAG
STA MAG PHASE
NO. oz-in deg
16 0.28 0
18 0.28 0
20 0.28 0
22 0.28 0
24 0.28 0
26 0.28 0
28 0.28 0
30 0.28 0